

Problem Set 1

General guidelines: This homework covers the material taught in the first two weeks of classes. It is due on October 9 at the beginning of class; no late submissions will be accepted. Please note that you should NOT work in groups on this homework; doing otherwise will be considered cheating. Finally, when submitting the homework, make sure to PRINT your name clearly on each page of the submission.

Problem 1 (10 points)

In class we defined the following simple grammar for arithmetic expressions with $+$, $-$, $*$ and $/$ operations:

$$\begin{aligned} E &\rightarrow T + E \mid T \\ T &\rightarrow F * T \mid F \\ F &\rightarrow 0 \mid 1 \mid \dots \mid 9 \mid (E) \end{aligned}$$

- a. Modify the grammar to include also an exponentiation operation \uparrow . The new operation should have highest precedence, and associate to the right.
- b. Give parse trees and leftmost derivations for the following strings:

$$\begin{aligned} &3 * 4 \uparrow 2 + 1 \\ &2 \uparrow (3 + 5) \uparrow 7 * 4 \end{aligned}$$

Problem 2 (10 points)

Consider the simple imperative language defined in class. We defined the operational semantics of the `for` V from E_1 to E_2 do S end

- a. Give an alternative semantics where expression E_2 is re-evaluated at each iteration.
- b. Give an example program that produces different results depending on the semantics of the `for` statement. Your program should terminate in both cases, but with different results.
- c. Describe the computations defined by the two operational semantics, showing that the final result is different.

Problem 3 (10 points)

- a. Use the axiomatic semantics of the simple imperative language we described in class to compute the weakest precondition of the following program fragment:

```
a := 2*b + 1;  
b := a - 3;  
{b < 0}
```

- b. The “division theorem” says that for every two integers $x \geq 0$ and $y > 0$, there exists two integers q (the *quotient*) and r (the *remainder*) such that $x = yq + r$ and $0 \leq r < y$. Prove the partial correctness of the following program to compute q and r using the axiomatic semantics. What can you say about its total correctness? Discuss partial correctness and total correctness if the precondition is weakened.

```
{x ≥ 0, y > 0}  
q:=0;  
r:=x;  
while (y ≤ r) do  
  begin r:=r-y;  
        q:=q+1;  
  end;  
{x = qy + r, 0 ≤ r < y}
```