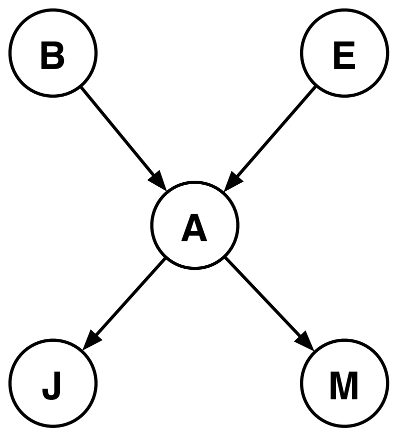
CSE 150. Assignment 2

Out: *Wed Apr 16*
Due: *Wed Apr 23*
Reading: Russell & Norvig, Chapter 14; Korb & Nicholson, Chapter 2.

2.1 Node ordering

Recall the alarm belief network (BN) described in class, with node ordering $\{B, E, A, J, M\}$ and directed acyclic graph (DAG) shown below:



Draw the (minimal) DAGs that would be required to represent this same joint distribution for the following alternative node orderings:

(a) $\{A, J, M, B, E\}$

(b) $\{A, B, J, E, M\}$

(c) $\{B, M, J, A, E\}$

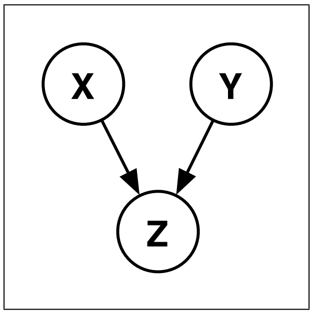
(d) $\{B, A, J, E, M\}$

(e) $\{J, A, B, E, M\}$

(f) $\{J, B, M, A, E\}$

It is **not** necessary to compute the CPTs in each of these alternative BNs. However, for each DAG, you should briefly justify the edges that you include or omit by appealing to properties of conditional independence.

2.2 Noisy-OR



Nodes: $X \in \{0,1\}$, $Y \in \{0,1\}$, $Z \in \{0,1\}$

Noisy-OR CPT: $P(Z=1|X,Y) = 1 - (1-p_x)^X \, (1-p_y)^Y$

Parameters: $p_x \in [0,1]$, $p_y \in [0,1]$, $p_x < p_y$

Suppose that the nodes in this network represent binary random variables and that the CPT for $P(Z|X,Y)$ is parameterized by a noisy-OR model, as shown above. Suppose also that

$$0 < P(X=1) < 1,$$
$$0 < P(Y=1) < 1,$$

while the parameters of the noisy-OR model satisfy:

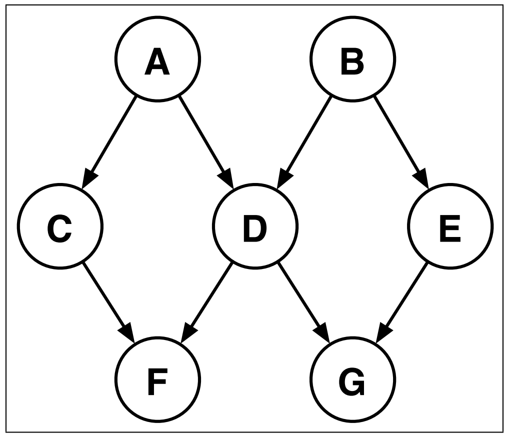
$$0 < p_x < p_y < 1.$$

Consider the following pairs of probabilities. In each case, indicate whether the probability on the left is equal $(=)$, greater than $(>)$, or less than $(<)$ the probability on the right. The first one has been filled in for you as an example. Briefly justify each answer.

	$P(X=1)$	$=$	$P(X=1)$
(a)	$P(Z=1|X=0,Y=0)$		$P(Z=1|X=0,Y=1)$
(b)	$P(Z=1|X=1,Y=0)$		$P(Z=1|X=0,Y=1)$
(c)	$P(Z=1|X=1,Y=0)$		$P(Z=1|X=1,Y=1)$
(d)	$P(X=1)$		$P(X=1|Y=1)$
(e)	$P(X=1)$		$P(X=1|Z=1)$
(f)	$P(X=1|Z=1)$		$P(X=1|Y=1,Z=1)$
(g)	$P(X=1)\,P(Y=1)\,P(Z=1)$		$P(X=1,Y=1,Z=1)$

2.3 Conditional independence

For the belief network shown below, indicate whether the following statements of conditional independence are **true (T)** or **false (F)**. Briefly justify each answer.



__________ $P(C, D|A) = P(C|A)\, P(D|A)$

__________ $P(A|D) = P(A|B, D)$

__________ $P(C, E) = P(C)\, P(E)$

__________ $P(C, D, E) = P(C)\, P(D)\, P(E)$

__________ $P(F, G) = P(F)\, P(G)$

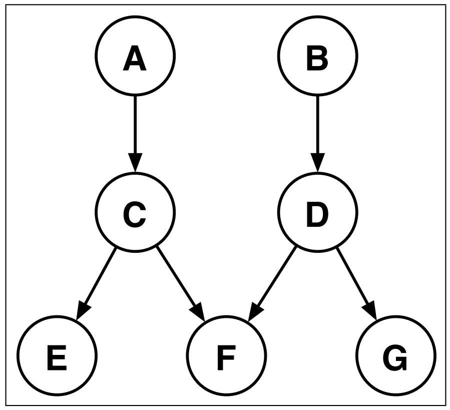
__________ $P(F, G|D) = P(F|D)\, P(G|D)$

__________ $P(A, D, G) = P(A)\, P(D|A)\, P(G|D)$

__________ $P(B|E) = P(B|E, G)$

__________ $P(C|E) = P(C|E, G)$

2.4 Markov blankets



For the above belief network, consider the following statements of conditional independence. Indicate the largest subset of nodes $\mathcal{S} \subset \{A, B, C, D, E, F, G\}$ for which each statement is true. Note that one possible answer is the empty set $\mathcal{S} = \emptyset$ or $\mathcal{S} = \{\}$ (whichever notation you prefer). The first two have been done as examples.

$$P(A) = P(A|\mathcal{S}) \qquad \mathcal{S} = \{B, D, G\}$$

$$P(A|C) = P(A|\mathcal{S}) \qquad \mathcal{S} = \{B, C, D, E, F, G\}$$

$$P(C) = P(C|\mathcal{S}) \qquad \_\_\_\_\_\_\_\_\_\_$$

$$P(C|A) = P(C|\mathcal{S}) \qquad \_\_\_\_\_\_\_\_\_\_$$

$$P(C|A, E) = P(C|\mathcal{S}) \qquad \_\_\_\_\_\_\_\_\_\_$$

$$P(C|A, E, F) = P(C|\mathcal{S}) \qquad \_\_\_\_\_\_\_\_\_\_$$

$$P(C|A, D, E, F) = P(C|\mathcal{S}) \qquad \_\_\_\_\_\_\_\_\_\_$$

$$P(F) = P(F|\mathcal{S}) \qquad \_\_\_\_\_\_\_\_\_\_$$

$$P(F|C) = P(F|\mathcal{S}) \qquad \_\_\_\_\_\_\_\_\_\_$$

$$P(F|C, D) = P(F|\mathcal{S}) \qquad \_\_\_\_\_\_\_\_\_\_$$

$$P(B, G) = P(B, G|\mathcal{S}) \qquad \_\_\_\_\_\_\_\_\_\_$$