Assignment 5

1. Consider a binary relation *Parent* where *Parent*(a, b) means that a is the parent of b. Assume that *Parent* is a forest (a union of trees). (i) Write an FO+μ^+ formula $\varphi(x, y)$ defining the set of pairs $< c, d >$ where c and d have a common ancestor and are of the same generation with respect to this ancestor (i.e. are at the same distance from their least common ancestor). (ii) Write an SO formula defining the same set.

2. ($\star$) Disprove the following statement: the properties expressible in *fixpoint* are exactly the PTIME properties that have a 0-1 law. **Hint:** Cook up a property P so that (i) P is in PTIME and is almost surely true, and (ii) if P is expressible in FO+μ^+ then *even* is also expressible in FO+μ^+.

3. Show that the *fixpoint* sentences collapse to FO on unary inputs. In other words, let R be a unary relation. For each sentence $\varphi \in$ FO + μ^+ over vocabulary $\{R\}$, there exists an equivalent FO sentence ψ over the same vocabulary.

4. ($\star$) Exhibit a logic that expresses exactly the PSPACE properties on finite structures (with no assumption on the presence of order).