

## Problem Set 4

These are practice problems on the subject of computability theory. In solving these problems you may use without proof the facts from the computability crib sheet, but you may not use any other facts without proof.

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**Problem 1** Prove that the following language is co-recognizable:

$$A = \{ \langle G_1, G_2 \rangle : G_1, G_2 \text{ are CFGs and } L(G_1) = \overline{L(G_2)} \} .$$

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**Problem 2** Prove that the following language is undecidable:

$$A = \{ \langle G_1, G_2 \rangle : G_1, G_2 \text{ are CFGs and } L(G_1) = \overline{L(G_2)} \} .$$

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**Problem 3** Prove that the following language is undecidable:

$$A = \{ \langle G_1, G_2, G \rangle : G_1, G_2, G \text{ are CFGs and } L(G) = L(G_1) \cap L(G_2) \} .$$

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**Problem 4** Prove that the following language is undecidable:

$$A = \{ \langle M_1, M_2 \rangle : M_1, M_2 \text{ are TMs and } M_1(\varepsilon) \text{ halts and } M_2(\varepsilon) \text{ loops} \} .$$

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