

How to Find Interesting Palindromes

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Abstract

Let Σ be a finite alphabet, $D \subseteq \Sigma^*$ be a finite dictionary, the elements of which we will call *words*. Let $n = \max\{|x| \mid x \in D\}$ be the size of a longest word. A *palindrome* is a nonempty string $x \in \Sigma^*$ such that $x = x^R$, where x^R is x written backwards. A *sentence* is a string x such that there exist words $w_1, \dots, w_k \in D$ with $x = w_1 \cdots w_k$. A palindrome is *interesting* iff it is a sentence.

We show (1) how to efficiently decide sentences, and therefore how to decide the interesting palindromes, and (2) how to generate an interesting palindrome. This is significant since, in light of Post's Correspondence Problem, it is not obvious that (whether a dictionary admits any interesting palindrome) is decidable. It is not clear to the author whether there is any *efficient* way to generate interesting palindromes.

1 Deciding sentences

A simple dynamic program taking a string x of size k as input can tell whether x is a sentence by constructing a bit vector $b[1, \dots, |x| + 1]$ and filling it from back to front so that $b[i] = 1$ iff the suffix $x_i \cdots x_k$ is a sentence.

Initially, $b[k + 1] = 1$ since the empty string is trivially a sentence. Then, supposing $b[i + 1], \dots, b[k + 1]$ are filled in correctly, $b[i] = 1$ iff there is a word $w \in D$ such that w is a prefix of $x_i \cdots x_k$ and $b[i + |w|] = 1$. The whole algorithm takes time $O(kn|D|)$. This bound could probably be tightened a lot by using better data structures to access the dictionary, but we won't bother with that here.

2 Generating an interesting palindrome

First let us rule out a simple case: if there is some $w \in D$ and $i \in \{0, \dots, |w| - 1\}$ such that $ww_i \cdots w_2w_1$ is an interesting palindrome then we are done. So suppose there are no such w, i .

We generalize the problem to the problem of, given a fixed string y , finding a short (but not necessarily shortest) positive length sequence of words x such that xy is a palindrome.

We can solve the generalized problem with a backtracking algorithm. At each step of the algorithm, we keep the set S of those suffixes y of length $\leq n$ for which we have already attempted to find a sequence x of words such that xy is a palindrome. Initially, $S = \emptyset$ and we try to find a sequence of words x for which xy is a palindrome where $y = \epsilon$ is the empty string.

To solve the general case, we first choose a word $w \in D$. Since we ruled out the trivial case in the first paragraph, if w begins any short palindrome xy where x is a sentence, then either $|w| \geq |y|$ and y is a suffix of w^R , in which case there is a short palindrome $x'y'$ where x' is a sentence and y' is y but with the trailing w^R stripped off; or $|w| < |y|$ and there is a sequence of words z_1, \dots, z_k such that

- both w^R and y are suffixes of $z_1 \cdots z_k$
- y is not a suffix of $z_2 \cdots z_k$
- there is a short palindrome $x'y'$ where x' is a sentence and y' is $z_1 \cdots z_k$ but with the trailing y stripped off.

Notice that in the recursion there is no point in looking for some sentence x' to prefix y if we already made a call looking for a sentence x to prefix this same y earlier in the call tree. This is because even if there were such an x' , we have proven that using it will not generate a shortest x , and that another branch will find a shorter x if one exists. We keep track of which calls were made with the set S .

Notice that we do not necessarily generate shortest interesting palindromes, but once we are able to prove that some other branch can generate a shorter interesting palindrome (if there is one at all), we immediately short circuit.

The branching factor for the call tree is $\leq |D|^2 2^n$. To see this, note that we choose a word w , then each position in y either starts a new word or not, then we choose a word z_1 to straddle the left boundary of y . The depth of the call tree is $\leq |\Sigma|^n$ since this is the maximum possible size of S .

So a shortest interesting palindrome has size at most $2n|\Sigma|^n$.

3 Some palindromes

I'm always amazed at how people find palindromes like these:

- T. Eliot, top bard, notes putrid tang emanating, is sad. I'd assign it a name: gnat dirt upset on drab pot-toilet.
- On a clover, if alive, erupts a vast, pure evil; a fire volcano.
- Do good? I? No! Evil anon I deliver. I maim nine more hero-men in Saginaw, sanitary sword a-tuck, Carol, I – lo! – rack, cut a drowsy rat in Aswan. I gas nine more hero-men in Miami. Reviled, I (Nona) live on. I do, O God!