Section Notes

October 17, 2012

1 Agenda

1. Sigmoid function
2. Conditional independence and d-seperation

2 Sigmoid function

2.1 What is it?

$$\sigma(z) = \frac{1}{1+e^{-z}}$$

2.2 Some useful properties

1. $\sigma'(z) = \sigma(z) \cdot \sigma(-z)$
2. $\sigma(z) + \sigma(-z) = 1$

2.3 Hints for logistic regression

We know that

$$\begin{aligned} P(Y=1|X_1=x_1,...,X_k=x_k) &= P(Y=1|\overrightarrow{X}=\overrightarrow{x}) \\ &= \sigma(\overrightarrow{w}.\overrightarrow{x}) \end{aligned}$$

We want to take the partial derivative of $\log P(Y=y|\overrightarrow{x})$. Here is a hint that could help you significantly. In your homework, you need to do some work to justify this hint based on facts from previous sections. Do not use this hint as a fact.

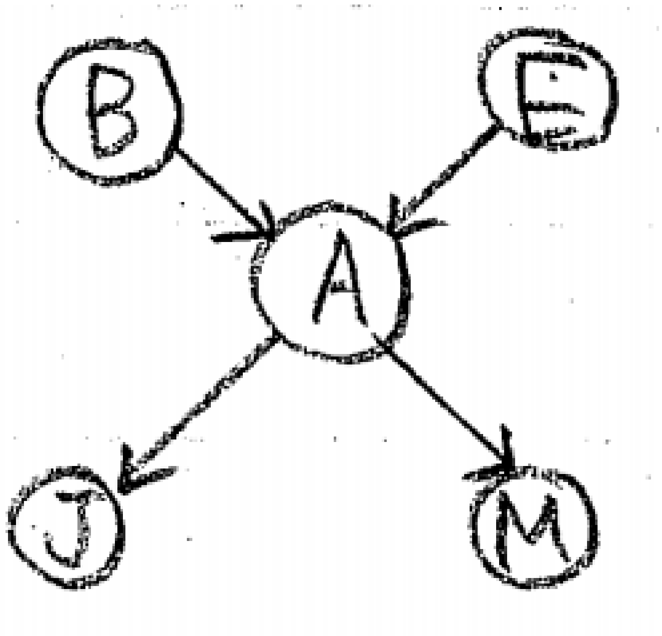

Figure 1: Exercise 1

$$\begin{aligned}\log P(Y=y|\overrightarrow{x}) &= \log[\sigma(\overrightarrow{w}.\overrightarrow{x})^y\sigma(-\overrightarrow{w}.\overrightarrow{x})^{1-y}] \\ &= y\log(\sigma(\overrightarrow{w}.\overrightarrow{x})) + (1-y)\log\sigma(-\overrightarrow{w}.\overrightarrow{x})\end{aligned}$$

Your final answer of the partial derivative should be simple and make sense.

3 Conditional independence and d-separation

Please refer to handout(http://www.pxnguyen.com/files/dseparation.pdf) for the notes on conditional independence and d-separation.

3.1 Example 1

To answer question in this section, please refer to the model in Figure 1. State whether the following statements are true or false.

1. $P(B|A,M) = P(B|A)$
2. $P(J,M|A) = P(J|A)$

3. $P(B, E) = P(B)P(E)$

4. $P(B, E|A) = P(B|A)P(E|A)$

Solutions:

1. $P(B|A, M) = P(B|A)$. True, A is the intervening event

2. $P(J, M|A) = P(J|A)$. True, A is the common cause.

3. $P(B, E) = P(B)P(E)$. True, A is the common effect.

4. $P(B, E|A) = P(B|A)P(E|A)$. False, the path B-A-E fails all three conditions.

3.2 Example 2

To answer question in this section, please refer to the model in Figure 2. State whether the following statements are true or false.

1. $P(F|C) = P(F)$

2. $P(F|C, E) = P(F|E)$

3. $P(A|C, G) = P(A|G)$

Solutions:

1. $P(F|C) = P(F)$. True

2. $P(F|C, E) = P(F|E)$. False. The path F-D-B-E-C doesn't hold for all three conditions.

3. $P(A|C, G) = P(A|G)$. False. The path A-D-F-G-H-E-C doesn't hold for all three conditions

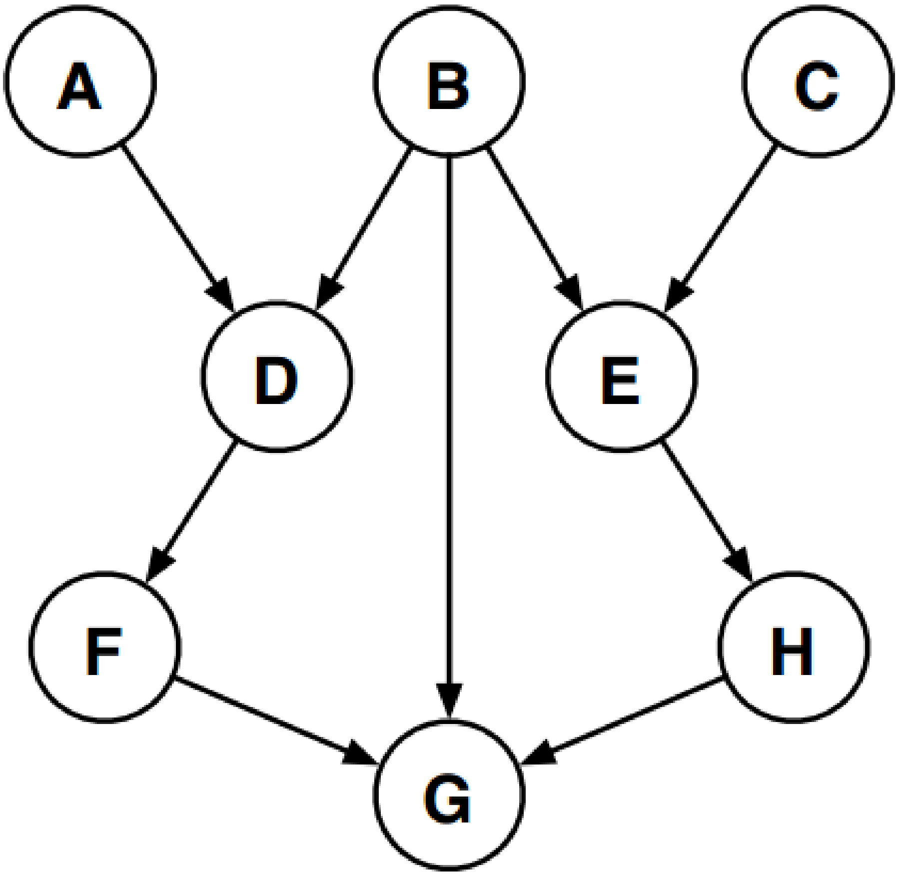

Figure 2: Exercise 2