

Mathematical Models of Cognitive Space and Time

Joseph Goguen
UCSD

Q. Introduction & Motivation

[See the abstract]

Work of Eleanor Rosch on concepts:

Experiments show prototype effects:

Can ask "how much is a penguin like a bird? [0,1,2,3,4]"

Also show basic level concepts:

- mid level of concept hierarchy
- shortest names
- seen as gestalts
- most associated knowledge
- learned earlier
- easier to learn

Work of George Lakoff on metaphor:

- metaphors come in families, called basic image schemas, sharing a sensory-motor pattern
- EX: MORE IS UP (you have to look higher)
 - "That raised his prestige"
 - "The market is up"
 - "This is a high stakes game"
 - and many more...
- Are also many other such schemas

Lakoff sees metaphors as maps of conceptual spaces:

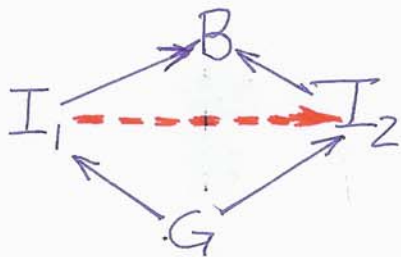
Ex: "My love is a rose"

Uses "rose" space concepts to enrich "mylove" space concepts.

Refers to 1985 Fauconnier book Mental Spaces for spaces & maps.

The blending theory of Fauconnier & Turner improves on this:

A metaphor is a blend of 2 spaces & the map is a "side effect" of that.



"Generic space" G is shared by I_1, I_2 input spaces.

Importation of content from I_1 is selective:
perfume, but not insects, fertilizer

Because image schemas grounded in experience, space & motion are important.

There are many examples, some theory,
but is not very clear what is possible & what is not.

There is no fundamental theory, that could do that

Time as space is of particular interest.

"I look forward to that"

"It was long ago"

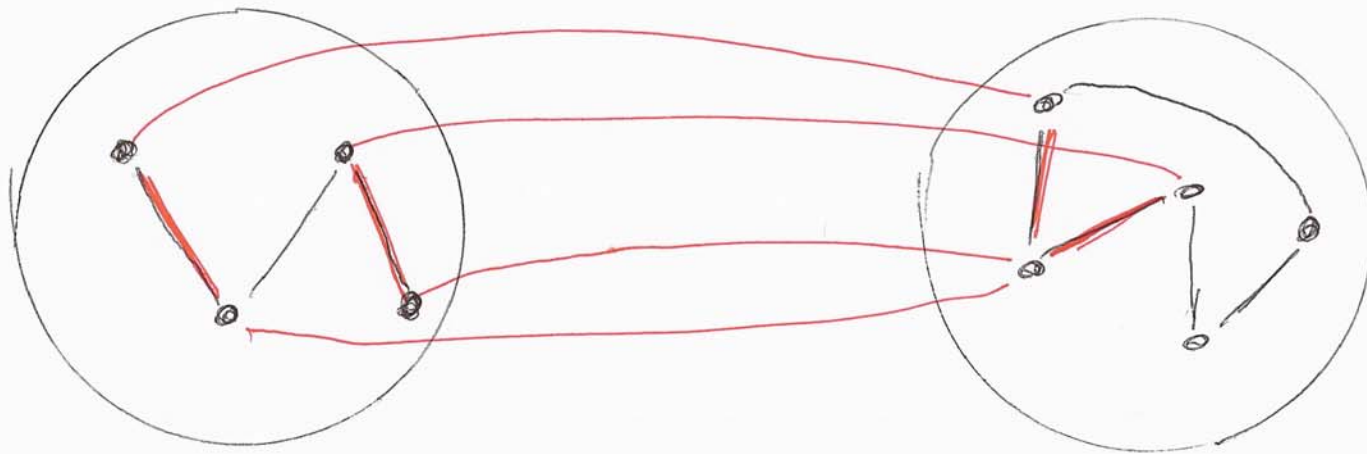
..... Much more

1. Foundations & Methodology.

Fauconnier's mental spaces — now called conceptual spaces — from a logical view, consist of constants and relations instantiated with these constants.

Conceptual maps take (some) constants and relations in one space to others in another space.

Can draw some nice pictures:



Cognitive linguists rarely use formalism — they use the pictures & of course linguistic data.

Allergic to formalism?

Bad academic politics, history; different education; ...

Chomsky, Montague, ...

A Newtonian Analogy

Calculus is a language; gravitation is a physical theory.

The language is used to express the theory —

- it helps make assumptions explicit
- it allows exploring consequences of assumptions
- it makes reasoning more reliable
- it makes theories more explicit
- & therefore more falsifiable.

Many theories can be expressed in the calculus:

- Some may be special cases of others
- Some may be inconsistent with others
- or even with themselves!
- some may be equivalent to others
- Can eliminate those inconsistent with data
- Can look for simplest formulation
- Can look for simple approximations —
can sometimes be more useful!

All this suggests a pluralistic, pragmatic approach, NOT a unitary dogmatic approach.

Mathematical theories are models, built for particular tasks

Can be many models, since there are many tasks.

Of course, some models are better: simpler, more general, more exact, ...

But one does not need to assume realism & reductionism:

a good model is not an ultimate truth!

Of course, no claim is made that the nascent language that I suggest will have anything like the impact of the calculus!

For one thing, we are at much too early a stage.

Some ideas are the following:

- semiotik spaces & semiotik morphisms - generalize conceptual spaces:
 - functions & terms
 - types
 - axioms
- frames, to express relations between concepts & percepts
"the symbol grounding problem"
- geometrical models as spaces of percepts
"conceptual spaces" in the sense of Gärdenfors
- blending of frames, which are satisfaction relations between semiotik spaces and geometrical spaces -

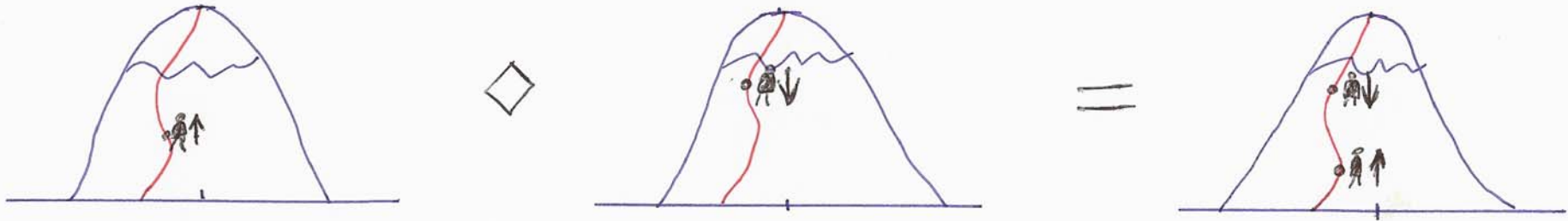
Within cognitive linguistics/semantics, a new trend is to study gesture as a way to ground image schemas, metaphors, blends, etc. in visible experiential reality - & even controlled experiments.

Intended to meet objections that evidence is weak, theories are conjectural, etc.

Note that all this begins to involve geometry in a real way - as did Newton!

2. A Buddhist Monk Meets Himself

[Read the Puzzle]



We will give 4 models for this.

Warning: We will also use the word "model" in 2 ways.

Can't do this in logic only need "models" of the logic - & there can be many of them.

Here are the theories:

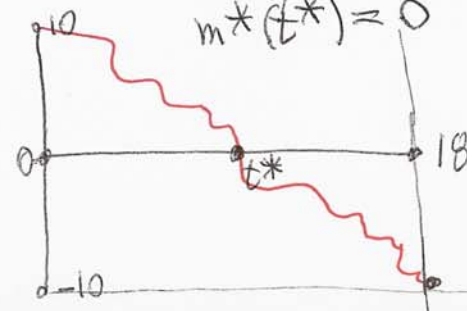
$$\begin{aligned} \text{Time} &= [6, 18] \\ \text{Loc} &= [0, 10] \\ m: \text{Time} &\rightarrow \text{Loc} \\ m(6) &= 0 \\ m(18) &= 10 \\ (\forall t, t': \text{Time}) t > t' &\Rightarrow m(t) > m(t') \end{aligned}$$

$$\begin{aligned} \text{Time} &= [6, 18] \\ \text{Loc} &= [0, 10] \\ m: \text{Time} &\rightarrow \text{Loc} \\ m(6) &= 10 \\ m(18) &= 0 \\ (\forall t, t': \text{Time}) t > t' &\Rightarrow m(t) < m(t') \end{aligned}$$

$$\begin{aligned} \text{Time} &= [6, 18] \quad \text{Loc} = [0, 10] \\ m, m', m^*: \text{Time} &\rightarrow \text{Loc} \\ t^*: \text{Time} & \\ m(6) = 0 \quad m(18) = 10 & \\ m'(6) = 10 \quad m'(18) = 0 & \\ (\forall t, t': \text{Time}) t > t' &\Rightarrow m(t) > m(t') \\ (\forall t, t': \text{Time}) t > t' &\Rightarrow m'(t) < m'(t') \\ m^*(t) &= m'(6) - m(t) \\ m^*(t^*) &= 0 \end{aligned}$$

Strict monotone intermediate value theorem implies:

$$\exists t^* : m^*(t^*) = 0$$



This is an amazing blend where 1 monk splits into 2 & 2 days merge into 1!

Strict monotonicity is too strong — but implicitly assumed by Fauconnier & Turner.

Can we weaken monotonicity:

$$(\forall t, t': \text{Time}) t > t' \implies m(t) \geq m(t')$$

Now the monk can stop & smell the flowers —

and he can meet himself over any $I \subset (0, 18)$! [I is closed]

This result surprised F & T.

But can go further: just assume continuity only:

This uses the most classical Intermediate Value Theorem of classical real analysis. Now the monk can meet himself over any number of disjoint closed subintervals!

This is a bit unrealistic, so can put a bound on velocity, say $|m'(t)| \leq 1$.

Can also consider bounds on $\frac{d^2 m}{dt^2}$ &c.

Many different models for monk movement!

It is interesting to explore & compare them.

Models in the other sense: functions $m: [0, 18] \rightarrow [0, 10]$ that satisfy the axioms: $m \models T$ & $m' \models T'$ & $m^* \models T^*$

Also have: T_1, T_2, T_3, T_4 ; T'_1, T'_2, T'_3, T'_4 ; $T_i^* = T_i \diamond T'_i$.

Let $M_i = \{m \mid m \models T_i\}$, $M'_i = \{m' \mid m' \models T'_i\}$, $M_i^* = \{m, m', m^* \mid \models T_i^*\}$

Say a frame is signature Σ , Σ -theory T , Σ -models m , Σ -satisfaction \models_Σ .

Are really blending frames here:

$\Sigma =$ "context", theories logical, models geometric.

2.1 Philosophical Sidebar

Frames are a Peircean triad:

Peirce advocated a triadic theory of meaning — as opposed to dyadic denotational semantics.

Such a theory is not realist, not relativist (not idealist, not nominalist)
IMHO, these are two foolish extremes.

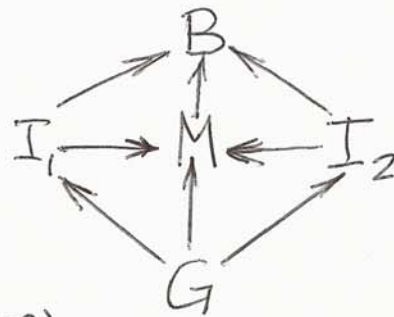
Idea is to unify/synthesize the extremes: concept & percept, subject & object

This is an aspect of Peirce's pragmatism.

2.2 Emergent Structure & Creativity

The monk blend actually involves recruiting a new space for "meeting":

$$\begin{aligned} a, b, d: \text{Time} \rightarrow \text{Loc} \\ t^*: \text{Time} \\ (\forall t: \text{Time}) d(t) = a(t) - b(t) \\ d(t^*) = 0 \end{aligned}$$

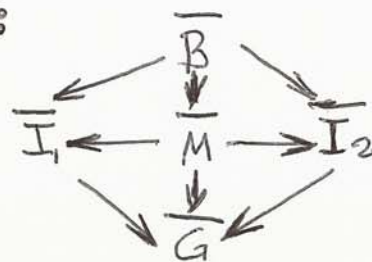


This is a conceptual integration network

B is a kind of colimit?

G has no monotone axioms, only continuity (or C^∞)

For the model spaces, get a dual diagram:



projection maps

\bar{B} is a limit

This is where creativity comes in:
find a space that "matches" existing spaces
& "completes the picture"

— Does not "solve" creativity but perhaps clarifies it a bit.

Big Claim [F&T]: Conceptual integration underlies all human cognition.

Note that a primary purpose of conceptual maps is to move inferences from source to target.

Note that we are really blending frames not just conceptual spaces.

Need optimality principles to determine what to include (partial maps)

"Disoptimality" principles for novel poetic blends

I am withered, impervious, like a swan of felt. — Neruda

cheap winter hats of fate. — Rilke

Dynamic HCI blend: window & scrollbar:

"Semiotic morphisms, blending & user interface
Design" [Goguen]



3. Time Flies

[Read Basho translation]

Basho, Oku-no-hosomichi [1690-94]

Narrow Road into the Interior

(Trans. Sam Hamill (+J. Goguen))

There is massive cross cultural evidence that time is primarily conceptualized as space.

In all languages so far, temporal vocabulary is primarily spatial.

[Niñez & Sweetser (2005)]

Example: The meeting scheduled for Wednesday has been moved forward two days.

Question: Is it Monday? Or Friday?

Most general metaphor is Space \Rightarrow Time. Two specializations:

(1) Time passing is motion over landscape. [Moving Time]

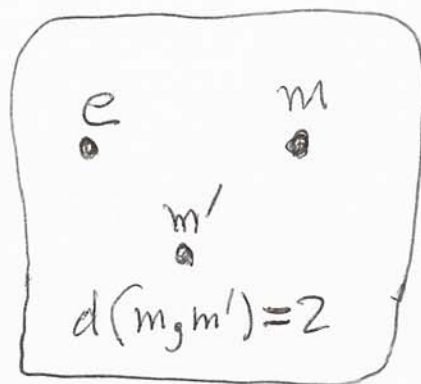
(2) Time passing is motion of an object [Moving Ego]

] Lakoff & Johnson (1999)

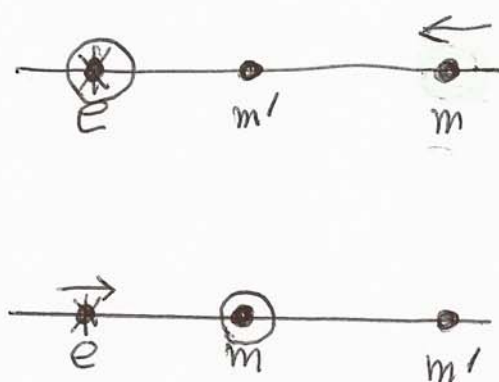
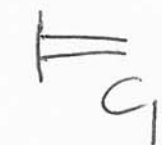
In (1) Fixed Ego, in (2) Fixed temporal reference.

Examples: (1) The end of the year is approaching.
(2) We are coming to the end of the year.

General Rule: Can't have both ego & time moving.



Theory



Models

} different meaning of "forward"

Notation: * = Ego

⊙ = Ref Pt. (Landmark, background)

→ = Moving

⊠ = Focus (foreground, trajectory)

Under (1) "forward" is relative to "front" of $m = \text{Wednesday}$ (moving towards Ego)

Under (2) "forward" is relative to "front" of $e = \text{Ego}$ (moving towards $m = \text{landmark}$)

So (1) $\Rightarrow m' = \text{Monday}$

(2) $\Rightarrow m' = \text{Friday}$

Note that "front" is also metaphor, determined by direction of motion.

Space as time metaphors can also be static:

Example: The due dates are too close together.

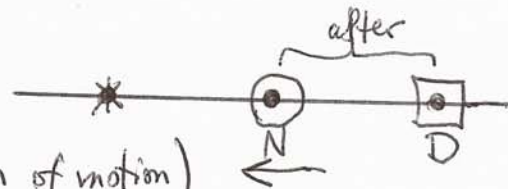
Here times are just locations in space.

Times can be moving wrt other times

Example: December follows November.

Nov is landmark, moving to Ego.

Dec follows after it (rear of direction of motion)



Can have blends involving times:

Example: Time flies like an arrow.

Blend gives velocity to time.

There are three dualities:

- (1) Ego Ref / Time Ref
- (2) Static / Dynamic
- (3) Trajectory / Landmark

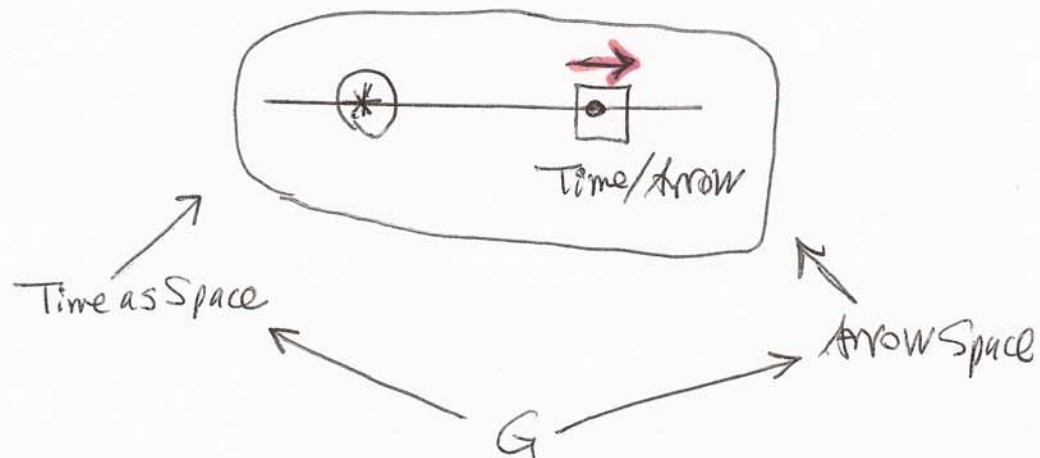
So things can be very complex!

The pot was further stirred up by Aymara:

where there is an unusual static metaphor of time as space, where the future is behind Ego-Ref pt.

[See lovely Núñez & Sweetser paper]

Evidence from gesture:
videos, even control groups
with Spanish & bilingual



For all dynamic cases, future is in front of Ego-Ref:

Experiencer faces the flow of events.

Note that past & future are inherently deictic [= wrt Ego-Ref]

(like here & there, or near & far)

Not wrt other times

So "20 mins before noon"

"Xmas follows Thanksgiving"

are not counter examples.

3.1 Further Complications

Most literature considers 2-ary relations:

before, after, follows

- Could also consider 3-ary:
between

Example: When did you lose your wallet?

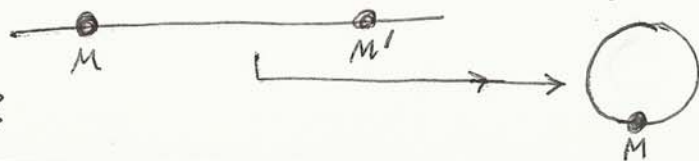
It was between Venice and Verona.

- Ego could be plural:

We bought our tickets before and after the noon deadline.

- Time can be cyclic:

He didn't think about taxes again until March came around again.



Malagasy. Mayan?

- Branching time
- Parallel universes

Sounds like SF (and could be!) but also:

- counterfactual situations
- fictive situations

Can violate transitivity of "after"

Can even violate Archimedean axiom:

$$(\forall a, b) a \neq b \implies (\exists c) a < c < b \text{ or } b < c < a$$

- Time travel:

We've got to go back to the future again.

[Back to the Future, movie series]

Time line for each individual - can intersect in multiple ways.

- Haiku: can induce experience of time stopping at "cut" word of blend.

[Hiraga]

Amazing thing is that we can easily understand these!

4. Conclusions

Monk example shows that math modelling can yield surprising insights, especially about implicit assumptions & their consequences.

Also shows needs:

- types
- geometric models
- satisfaction & context

Frames & frame blending are really new ideas seem to have a lot of potential:

- symbol grounding
- concept/percept, subject/object

Meeting example shows can classify possibilities in simple cases:
i.e., can reason about possible models

Goal: Classify all possible humanly reasonable Space \Rightarrow Time metaphors

Other examples show that this can be very complicated!

But we seem to have the machinery to make some sense of it.

Seem to be some general rules:

- not both Ego & Time can be moving

Also some (modest) insights into creativity

- Novel optimality principles for novel blends
e.g., contemporary poetry.

Cognitive linguistics/semantics/psychology gives fascinating insights into what it means to be human:

- "neo-Kantian categories": what is truly basic?
What properties are important?
→ Space is more basic than time.
- What values are truly basic?
- What is cognition?

Hope of transcending many old & silly dichotomies & controversies.

- idealism/nominalism
- nature/nurture
- ...
- What is creativity?
- How does language work to create meaning?
- What is meaning?